TOPOLOGY-IV
M.MATH-II
FINAL EXAM

Total 50 Marks

(1) Let M be a compact manifold with trivial tangent bundle. Show that $\chi(M) = 0$.
(5 marks)

(2) Prove that there is no nowhere vanishing vector field on S^{2n}. (6 marks)

(3) If M is a compact manifold of odd dimension then show that the Euler characteristic $\chi(M) = 0$.
(10 marks)

(4) Show that $H^p(S^2 \times S^4) \cong H^p(\mathbb{CP}^3)$ for every p, but that the graded algebras $H^*(S^2 \times S^4)$ and $H^*(\mathbb{CP}^3)$ are not isomorphic. (5 marks)

(5) If M is an oriented connected manifold of dimension n, then compute $H^n(M)$. (8 marks)

(6) Let M^m be a compact smooth submanifold of S^n with $0 < m < n-1$. Let $U = S^n - M^m$. Construct isomorphisms

$$H^p(U) \cong H^{n-p-1}(M)^* \qquad (1 \leq p \leq n-2)$$

and show that $H^p(U) = 0$ for $p \geq n-1$ (8 marks)

(7) Consider a smooth map $f : N \to M$ between n-dimensional oriented smooth connected compact manifolds. Prove that if $H^n(f) : H^n(M) \to H^n(N)$ is non-zero, then $H^p(f) : H^p(M) \to H^p(N)$ is injective for every p. (8 marks)